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ELASTIC PLATES AND SHELLS AND THE STABILITY OF THIN-WALLED STRU--ETC(U)  
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ELASTIC PLATES AND SHELLS AND STABILITY  
OF THIN-WALLED STRUCTURES

Grant # AFOSR 71-2063

FINAL SCIENTIFIC REPORT

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From the beginning of the initial grant period, the principal investigator focused on what seemed to him certain fundamental phenomena of such a nature that any insights into them, including more powerful methods of solution even in special cases, would be of long term value in increasing our understanding of the whole area and ultimately in making more sophisticated calculations.

(i) Foremost among these was the deep buckling and the extended post-buckling regime for non-shallow shells, in particular, complete shells, of which the spherical shells and rotationally symmetric ellipsoidal shells are significant examples.

(ii) The literature indicated that a particularly powerful approach when the shell admits it is the formulation of approximate non-linear equations modeled after the Foppl-von Karman equations of plate theory, i.e., a pair of coupled non-linear partial differential equations of definite structure, namely with a highest order part linear and quadratic non-linear terms of lower order, for two unknowns, a displacement function and a stress function.

(iii) For the particular examples to be studied under (i) and which could be described by equations as in (ii) the principal investigator found that a particularly useful method of solution of the equations is the Galerkin or spectral method of expansion of the unknown functions in suitable orthogonal series. This method has the virtue of simplicity of principle and implementation together with the "modal" insight into the structure of solutions analogous to that obtained from orthogonal expansions of more traditional linear problems.

This, then, was the principal investigator's package: extended post-buckling analyses of non-shallow shells by means of orthogonal expansion solutions of von Karman type non-linear shell equations. After



some tests on plate problems in items (2,3,5) of the special bibliography (section 3) and an initial sketch of an attack on the spherical shell in item (6) some definitive progress is reported in item (10), which uses items (8) and (9) as computer programs, for the spherical shell and item (4).

Item (4) contains the first results of its kind (deep buckling of ellipsoidal shells) in the literature. Item (10) contains new results in the form of a study of deep buckling for much thinner spherical shells (radius-thickness ratio of 200) than in the literature and an unanticipated result - an indication of an asymptotic shape of the buckled shell at the lower critical pressure as the radius-thickness ratio tends to infinity. Besides these results perhaps the most significant result in (8,9,10) is the program SPHERE, which solves the very sensitive and ill-conditioned set of non-linear algebraic equations resulting from the Galerkin method even on the descending part of the pressure deflection graph, something that earlier investigators, for example, Hoff and coworkers were unable to do when applying the Galerkin method with trigonometrical series to the Donnell-von Karman equations for buckling of circular cylinders under axial compression. Further extensions of this work including the derivation of more accurate equations under heading (ii) have been achieved but not in time for written reports before the end of this grant.

One aim of the work done under heading (iii) was a more imaginative and flexible use of orthogonal expansions in applied analysis. A small part of what has been discovered is indicated in item (7).

Finally some contributions of some significance to the classical theory of hydrodynamic instability are reported in items (1), which uses the spectral method, and (11), the latter connected with low-speed aerodynamic noise.

2. Personnel supported (this list is compiled from memory and should be checked with earlier yearly reports filed for this grant).

Arlene Hoffman

Wilfredo Torres

Farley Mawyer

Fred Buckley

Johny Hellmann

Neal Jacobs

Terence Coffee

Jonathan Marz

James Korenthal

Michael Ecker

Frances Jackson

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#### 4. Special bibliography.

1. Coffee, Terence, The stability of plane Couette flow, AFOSR report, 1976.
2. Hellmann, Johnny, The non-linear bending of a clamped circular plate under uniform normal pressure, AFOSR report, 1975.
3. Hoffman, Arlene M., Non-linear large deflection bending of a clamped square plate under uniform normal pressure, AFOSR report, 1974.
4. Jacobs, Neal, Deep buckling of a thin oblate spheroidal shell under uniform normal pressure, AFOSR report, 1976.
5. Rauch, Harry E., Buckling of a circular plate under edge compression, Differential Geometry, in honor of K. Yano, Kinokuniya, Tokyo, 1972, pp. 415-422, AFOSR report, 1972.
6. \_\_\_\_\_, Instability of thin-walled spherical structures under external pressure, Contributions to Analysis, Ed. L.V. Ahlfors, Academic Press, New York, 1974, pp. 357-373, AFOSR report, 1974.
7. \_\_\_\_\_, Expansions in Legendre eigenfunctions of integral degree of Legendre eigenfunctions of non-integral degree arising in the study of incomplete spherical structures, AFOSR SCIENTIFIC REPORT, November, 1974.
8. \_\_\_\_\_, Jacobs, Neal H., and Marz, Jonathan L., SPHERE, a program for computing buckling loads of spherical shells under uniform pressure, AFOSR report, December 1975.
9. \_\_\_\_\_, TABLE, a program to compute  $\int_{-1}^1 (x/\sqrt{1-x^2}) P_\ell^1 P_m^1 P_n^1 dx$ , and the table thereof for indices,  $1 \leq \ell, m, n \leq 40$ , AFOSR report, December 1975.
10. \_\_\_\_\_, Budding of a complete spherical shell under uniform normal pressure,
11. Rauch, Harry E., Jet driven instruments and the edge-tone, AFOSR report, January 1976.



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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  <p>➔ Progress on bending and buckling of plates and deep buckling and post-buckling regime analysis for complete spherical and oblate spheroidal shells is reported. The use of von Karman-type equations and the Galerkin method (with orthogonal series) is reported. Some new work on orthogonal expansions and hydrodynamic stability are also reported. Special emphasis is placed on new program for solving non-linear numerical equations in several variables.</p>		